PHIL 470: Gödel's Incompleteness Theorems

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Semester:	Spring 2015
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Course Website:	myelms.umd.edu/courses/1133215
Office:	Skinner 1103A
Office Hours:	W 2 - 3.30 PM
Class Times:	TuTh 12:30pm - 1:45pm
Class Location:	LeFrak Hall 1201

Course Description

This course will focus on two famous theorems due to Kurt Gödel: The Incompleteness Theorems. The first theorem states, roughly, that every formal mathematical theory, provided it is sufficiently expressive and free from contradictions, is incomplete in the sense that there are always statements (in fact, true statements) in the language of the theory which the theory cannot prove. We will prove the 1st and 2nd Incompleteness Theorems and survey their technical and philosophical repercussions. In order to prove the Incompleteness Theorem(s), we will need to study the expressive power of formal languages and axiomatic theories and also discuss different approaches to effective computation: recursive functions, register machines, and Turing machines. We will discuss their equivalence, Church's thesis and elementary recursion theory.

Topics to be covered include: formal models of computation; Church's Thesis; Gödel's 1st and 2nd incompleteness theorems and their repercussions; Tarski's proof of the undefinability of truth; Undecidability of the Halting Problem; Decidable subsystems of arithmetic; provability logic (Kripke soundness and completeness, arithmetical soundness and completeness, fixed-point theorems), and The Knower Paradox (and epistemic arithmetic).

Prerequisites: PHIL370 (or equivalent logic course), or permission from the instructor.

Literature

The required texts for the course are:

1. P. Smith, *An Introduction to Gödel's Theorems* [IGT], Second Edition, Cambridge, 2013. (Available to buy in the bookstore)

I will also assign readings from the following texts (available on the course website):

- G. Boolos, J. Burgess, and R. Jeffrey, *Computability and Logic* [CL], Cambridge, 5th Edition, 2007.

- P. Raatikainen, "Gödel's Incompleteness Theorems", The Stanford Encyclopedia of Philosophy, http://plato.stanford.edu/entries/goedel-incompleteness/

The following texts are recommended for additional reading:

- H.-D. Ebbinghaus, J. Flum, and W. Thomas, *Mathematical Logic*, Springer, 1995.
- H. Enderton, *A Mathematical Introduction to Logic*, Academic Press, 2nd Edition, 2001.
- T. Franzen, *Gödel's Theorem: An Incomplete Guide to its Use and Abuse*, A K Peters, 2005.
- M. Fitting, *Incompleteness in the Land of Sets*, Kings College Press, 2007.
- R. Smullyan, *Gödel's Incompleteness Theorems*, Oxford Logic Guides, Oxford University Press, 1992.

Final Presentation

You will give an oral presentation on a topic of your choice related to material discussed in the course. Possible topics are listed on the course website. You must produce presentation notes that will be provided to the class during your presentation (these notes will be graded).

Grading Policy

The course requirements are: Participation & quizzes (10%), problem sets (30%), final presentation (30%), and a final exam (30%). I will periodically give quizzes (either in-class or online). The final will be an in-class exam given during finals week. For the problem sets, you are encouraged to work in small groups. You may discuss the problems with one another or with me as much as you want. *But you must always do the final write-up completely on you own.* A good strategy when working together is to use a blackboard and erase it completely before writing up your (separate) answers. Please write the name of your discussion partner(s) on the front page of your assignments. Solutions to the problem sets will be made available after the assignment is due and will be discussed in class. ***Late assignments will not be accepted for full credit.***

Tentative Syllabus

Below is a tentative schedule for the semester (consult the course site for more details).

Introduction to Incompleteness
Tue 1/27

- [IGT] Chapter 1
- Introduction to Incompleteness, Smullyan
- For historical and philosophical context: Kurt Gödel (SEP article), Hilbert's Program (SEP article)

Basic Concepts: Functions, enumerations, language of arithmetic, standard models of arithmetic, non-standard models of arithmetic, primitive recursive functions, recursive functions and sets, effective axiomatizations
Thu 1/29, Tue 2/3, Thu 2/5

- [IGT] Chapters 2 - 4
- [CL] Chapters 6.1, 6.2, 12, 13

Arithmetic definability of recursive functions
Tue 2/10, Thu 2/12

- [CL] Chapter 16.1
- [IGT] Chapter 15

Formal arithmetic: induction, BA, Q, $\mathsf{I}\Delta_0$, PA
Tue 2/17, Thu 2/19

- [IGT] Chapters 9 - 13

Capturing (representing) numerical properties, Q is $p.r$-adequate
Tue 2/24, Thu 2/26

- [IGT] Chapters 5, 16, 17
- [CL] 16.2

Arithmetization of syntax (Gödel numbering)
Tue 3/3, Thu 3/5, Tue 3/10

- [IGT] Chapters 19, 20
- [CL] 15.1 - 15.3

The First Incompleteness Theorem, Diagonal Lemma
Thu 3/12

- [IGT] Chapters 21 - 24
- Sep Article on Gödel's Incompleteness Theorem, Section 2

Spring Break: *Tue 3/17, Thu 3/19*

Recap of the First Incompleteness Theorem, Rosser's Theorem, Tarski's Theorem
Tue 3/24, Thu 3/26

- Sep Article on Gödel's Incompleteness Theorem, Section 2
- [IGT] Chapters 25, 27

The Second Incompleteness Theorem
Tue 3/31, Thu 4/2

- [IGT] Chapters 31, 32

Overspill Day: *Tue 4/7*

Implications — Truth vs. provability
Thu 4/9, Tue 4/14

- [IGT] Chapters 23, 37
- Sep article on Gödel's Incompleteness Theorem, Section 6

Löb's Theorem and Provability Logic
Thu 4/16, Tue 4/21, Thu 4/23

- [IGT] Chapters 33, 34
- [CL] Chapter 27

Additional topics: Montague and Kaplan's Knower Paradox, Second-Order Arithmetic, Turing Machines, Incompleteness and Undecidability, or Model-Theoretic Characterizations of PA
Tue 4/28, Thu 4/30

Student Presentations
Tue 5/5, Thu 5/7, Tue 5/12
