

explained to them on that basis may be an irrelevant empirical question. But it is an *a priori* constraint that it should at least be *possible* for someone to acquire their concepts in this way. For if this were not possible, it could only be because no one could understand the 'simpler' concepts who did not already understand the 'complex' ones, which would contradict the initial thesis about the manner in which they are embedded in one another.

Thus it may in fact be the case that people acquire their grasp of quantification – somewhat mysteriously – as a result of total immersion in a linguistic practice, rather than step by step on the basis of a prior grasp of singular propositions. Indeed, this is perhaps not implausible as a description of what actually happens. For one does not, in general, give children definitions, of any sort. One simply talks to them. (Compare 3.263.) All that Wittgenstein's argument requires is that it must be *possible* to acquire a grasp of the quantifiers on the basis of a prior understanding of propositions referring to Simples. And this will only be possible (given logical objectivism) if Simples do indeed exist.¹¹

Note also that to say, as Wittgenstein does, that all our propositions are actually truth-functions of elementary ones involving reference to Simples is not to say anything about what is going on in our thoughts, or about the manner in which the truth-conditions of our propositions are actually determined (that is to say, their senses). As we saw in Chapter 7, it is merely to claim that all propositions are logically equivalent to some possible truth-function of elementary ones. For recall from Chapter 1 that the *TLP* view is that the criterion of identity for semantic content is logical equivalence.

Summary

There is no obvious incoherence in the *TLP* doctrine of the existence of Simples. Yet there is a powerful argument in its support, premised upon the requirement of determinacy-in-advance (more generally, logical objectivism), which also provides us with the best available interpretation of the crucial passages in the text. It is therefore not unreasonable to regard Wittgenstein as having discovered a proof of the existence of Simples, in the absence of a direct refutation of logical objectivism.¹²

13 Independent elementary propositions

Our task in this chapter is to see why Wittgenstein should have insisted that the elementary propositions which constitute the end-point of analysis must be logically independent of one another.

(A) Preliminaries

The independence requirement is mentioned at four different points in *TLP*. Twice it occurs in the material mode, stated in terms of the independence of the states of affairs which elementary propositions describe (1.21, 2.061). Then at 4.211 it is said to be a mark of a proposition's being elementary that there can be no elementary proposition which contradicts it. Finally, 6.3751 makes essentially the same point in the context of a discussion of colour-exclusion. However, in none of these cases does a study of the surrounding remarks throw any light on Wittgenstein's motives. Nor can we look to *NB* for any explicit guidance, since the independence requirement receives no mention there. Yet he must surely have had some powerful reason for insisting upon it, since he is prepared to do so in advance of being able to give any examples of elementary propositions. Indeed, it is far from obvious that the requirement can possibly be complied with.

There is just one other remark in *TLP* which suggests a possible interpretation. This is 5.152, which tells us that any two elementary propositions will give one another a probability of $\frac{1}{2}$. When taken in the context of the theory of objective probability developed in the 5.15s, this implies that elementary propositions must be logically independent of one another.¹ Now, as Wittgenstein himself says, his theory of probability is founded on an identification of the trio 'necessary', 'possible' and 'impossible' with 'truth-functional tautology', 'truth-functional contingency' and 'truth-functional contradiction' respectively (4.464, 5.1), which itself pre-

supposes the independence requirement if taken with full generality. So if we could construct an argument supporting such an identification it would at the same time provide an explanation of the independence requirement.

(B) Displaying necessity

Throughout the 6.1s we find Wittgenstein explaining and arguing for his thesis that the propositions of logic are tautologies. Now by 'logic' here he does not just mean 'formal logic' – logic in the narrow sense; though this is certainly part of what he has in mind.² Rather, by a proposition of logic he means any conceptually necessary proposition. This is in line with the use of 'logic' and 'logical' throughout the rest of *TLP*, where he generally has in mind any internal or conceptual relationship (see, for example, 2.0121, 4.015, 4.023, 4.1213, and indeed the very title of *TLP* itself). In any case, if read in this light we can see the 6.1s as providing arguments for the thesis that all necessity is truth-functional, and hence as providing reasons for the independence requirement.

Wittgenstein insists that it must be possible to recognise a proposition of logic from the symbol alone – that is to say, from our knowledge of its sense (6.113, 6.126). In part this reiterates his view that logic is epistemologically prior to metaphysics, discussed in Chapters 2 and 3 above. Since it is unintelligible that we should have to read off our knowledge of the necessity of a proposition from a prior acquaintance with the metaphysical necessities which hold in the world, it must rather derive from seeing, from our knowledge of the senses of the terms involved, that the proposition is constructed in such a way as to be true come what may. But he also wants to insist upon something more: that it must be possible to *display* the sense of the necessary proposition in such a way that its status as necessary will be manifest.³ For 6.122 implies that it must be possible to construct notations in which we can discern that a proposition is necessary by mere inspection, as we can in the case of truth-tables.

We can begin to see what Wittgenstein has in mind here if we reflect upon another strand in the discussion in the 6.1s, namely his rejection of axiomatic approaches to logic (which now means 'logic' narrowly conceived of). He insists against Frege and Russell that there is a sense in which all the propositions of logic are on an equal footing, each one showing in and of itself that it is necessary (6.126, 6.127). Moreover, the role of proof in logic is merely psychological, enabling us to recognise necessary propositions in complicated cases (6.126). The claim is that the notion of proof

of a complex proposition from simpler cases should only enter into an account of the epistemology of logic, not into an account of what logic is.

Wittgenstein's remark at 6.1271, expressing surprise that a thinker 'as rigorous as Frege' should appeal to degree of self-evidence as the mark of a proposition of logic, is pertinent here. For it can indeed be said that for Frege a necessary truth gets characterised as one which follows from self-evident truths by means of self-evident principles of inference.⁴ But this is to introduce into the characterisation of logic just the sort of psychology which Frege himself is so resolute in rejecting elsewhere. For how can one continue to believe in the objectivity of logic – maintaining that a proposition's status as necessary is independent of facts about the human mind – while yet holding it to be constitutive of a truth of logic that it should either strike us as obvious, or be derivable from truths which strike us as obvious by means of principles of inference which strike us as obvious?

Since Wittgenstein, like Frege, is committed to the objectivity of logic as well as the epistemological priority of logic over metaphysics, he has no option but to say that logical truths hold in virtue of relations between concepts (senses), appealing to intuition only in accounting for our knowledge of what those concepts require of us in particular cases. But then since these relationships are objective, and not in any way constituted by intuitions, it must surely be possible to provide them with a semantic validation: setting out the relevant concepts in an analysis, in such a way that those relationships can be seen to depend only upon them. It is just such a validation that Wittgenstein is able to provide, in the case of logic narrowly conceived of, by means of his truth-table notation. This enables him to display a truth of logic in such a way that its truth manifestly depends only on the senses of the propositional connectives and their mode of combination.

Here is part of the reason, at least, why Wittgenstein should insist that the elementary propositions must be independent of one another. For if they were not, there would be a class of necessary truths which could not be exhibited as truth-functional tautologies; and yet how else could they receive their semantic validation? For example, if 'This is red' and 'This is green' were really elementary, then there would apparently be no way of explaining the necessary status of 'Not both: this is red and this is green'. But there must be something about the concepts *red* and *green* in virtue of which they exclude one another; their mutual exclusion cannot simply consist in the fact that anyone who grasps them intuitively rejects the admissibility of describing a surface as both 'red' and 'green' at once.⁵ Rather, such intuitions must reflect an awareness of the inner structure of

the concepts *red* and *green* themselves. Yet if 'This is red' were elementary the concept *red* would possess no inner structure. For to say that a proposition is elementary is precisely to say that the concepts involved in it admit of no further analysis.

This argument for the logical independence of elementary propositions thus turns fundamentally upon three premisses: logical objectivism, the epistemological priority of logic over metaphysics, and the plausible thought that if two propositions are inconsistent with one another, then this must reflect the structures (hidden or explicit) of those propositions themselves. Of course Wittgenstein was later to attempt to explain inconsistencies between elementary propositions by means of the idea of an holistic system of concepts, all of which are involved whenever any one of its members is applied to anything (thus denying the third of the above premisses – see *PR* 82–6). We shall return to this idea in Section D below. But first we shall explore a rather different way of approaching the independence requirement.

(C) Reducing necessity

Notice that on the *TLP* account a necessary truth is a truth about all possible worlds. For on the one hand, all necessary truths are characterised as truth-functional tautologies of elementary propositions (5, 5.1, 6.1). Yet on the other, to fix the sense of one elementary proposition is at the same time to fix the sense of all (5.524). Moreover, the elementary propositions can be used to describe reality completely: an assignment of truth-values to each one of them would constitute a complete description of the world (4.26). Indeed, since the objects with which these propositions deal are constituents of all possible worlds, all possible worlds may be described by means of such assignments (2.0124). Then since a tautology is a sentence which is true for all assignments of truth-values to its component elementary propositions, it will of course be true for all assignments of truth-values to the whole set of elementary propositions. So it will be a truth about all possible worlds.

Now, the important point here is this: because (and only because) the elementary propositions are logically independent of one another, Wittgenstein has to hand the materials for a reductive account of necessity. He can explain necessary truth as truth about all possible worlds (true for all assignments of truth-values to the set of elementary propositions) without taking for granted any modal notions. His account of the concept is then genuinely explanatory, in that it in no way presupposes what was to

be explained: the notion of necessity. Contrast with this any attempt to explain necessity in terms of a notion of a possible world not fixed by means of a set of mutually independent elementary propositions. For example, suppose that the language contains as elementary the sentence 'This surface is red' and 'This surface is green.' Then in order to recognise that an assignment of 'True' to both of them fails to constitute a part-description of a possible world, we should have to rely on our knowledge that the concepts *red* and *green* logically exclude one another. But this would be to take for granted the very notion of necessity we were supposed to be explaining.

It might be objected that Wittgenstein's account, too, presupposes an understanding of a modal notion, since it is given in terms of assignments of truth-values to a class of propositions characterised as logically independent. But here we need to distinguish between the fact that those propositions are logically independent of one another, on the one hand, and what would be required for an understanding of them on the other. Someone could surely have a complete grasp of their senses while having neither an implicit nor an explicit grasp of the notion of necessity. In contrast, no one could understand sentences involving the words 'red' and 'green' who was not prepared to reject *a priori* the admissibility of describing an object as being red and green all over. An ability to see the mutual incompatibility of the colour-terms is a criterion for their proper understanding. We are not required to see this as such, perhaps, since we may as yet have no word for 'incompatible'; but we must at least refuse to accept the description 'is red all over and green all over'. So anyone who understands a language containing such terms will already have an implicit grasp of the notion of necessity, in a way that someone who understands the sort of elementary propositions envisaged in *TLP* would not. Thus Wittgenstein's account – and only such an account – would put one in a position to explain the concept of necessity to someone completely *ab initio*.

It may be helpful at this point to compare the *TLP* account of necessity with the form of analysis provided in contemporary modal logic, specifically in Model Theory.⁶ This, too, takes as basic the idea of a set of atomic propositions in terms of which one could provide a complete description of reality; but without the requirement of logical independence. A 'model' for the language is then defined as being any sub-set of the set of all assignments of truth-values to the elementary propositions. The notion of necessity is then defined relative to a model, by saying that $\Box A = \text{True}$ for a given assignment of truth-values within a model, if and only if $A = \text{True}$ for every assignment in the model. Note that this account neither mentions

nor implicitly relies upon a prior grasp of the notion of necessity. For there is nothing to require that a model should be a set of assignments of truth-values to the atomics which is actually possible. Indeed, many models for a given language will in fact be impossible.

Someone could understand and work with the above definition of \Box who had no prior grasp of any modal notion. Given only a formal description of a language and a set of assignments of truth-values to its atomic sentences, they would be able to apply the definition. But for this very reason \Box must fail to capture our notion of necessity, since the two notions will not even be co-extensive. Thus $\Box A$ will come out as true relative to certain models where 'Necessarily A' would either be false or nonsensical. For example, \Box Mrs Thatcher is red all over and green all over' will be true relative to some models of English; yet it is hardly necessary. The best one could do would be to provide someone with an operator co-extensive with 'necessary' by giving the above definition of \Box and then presenting them with a model for English which is actually correct, which in fact describes the set of all possible worlds. But this could hardly succeed in conveying our notion of necessity. For there can be nothing in the explanation which picks out the model as special. So far as the trainee is concerned, it is simply one set of assignments of truth-values among others.

What this means is that contemporary Model Theory fails as a reductive account of necessity. This is not to say that it does nothing valuable. On the contrary, it can be seen as explicating a large part of our conception of necessity. It also has the great advantage of enabling us to use our firmly entrenched understanding of the rules governing the existential and universal quantifiers in working out the principles which govern necessity. (For example, the move from $\Box A$ to $\Diamond A$ gets validated as the move from 'True for all assignments in the model' to 'True for some assignment in the model'.) But the *TLP* account would have all of these advantages as well as providing us with a reductive analysis.

Wittgenstein's independence requirement would then be very attractive if it could be complied with; but we have yet to show that it must be possible to comply with it. The fact that the existence of a class of logically independent elementary propositions would enable one to give an *ab initio* explanation of necessity provides us with no reason to think that there must be such a class, unless there is some reason for thinking that it must be possible to give such an explanation. So the real question which needs to be asked is what reason there is for thinking that an eliminative definition of necessity is even so much as possible.

We might wonder whether the motivation behind a reductive account is a

ontological. Might Wittgenstein's demand be to show that necessity is not an element of the real world, just as he wishes to show that the logical constants (*and*, *or* and the rest) do not belong to the world? Is the idea that a complete but a-modal description of reality would show that modality is not genuinely there in the world? This is not a very promising interpretation. For on no account of the matter does 'necessary' function as the name of a supposed kind of thing. It is more naturally treated as a predicate of sentences. And as we saw in Chapter 1, Wittgenstein in any case wishes to deny that predicates have reference.⁷

Nor, on the face of it, does it appear very plausible to suggest that the line of thought is this: if it were to prove impossible to give a genuinely eliminative account of necessity, then one could never explain that concept to someone who had not already grasped it. For the natural response to this is precisely to deny that any concept-user could ever be without a grasp of necessity. On the contrary, perhaps that concept must be implicit in a grasp of concepts from the start; perhaps it is a primitive concept, having to be introduced alongside the other basic concepts that a person learns. But on reflection this natural response turns out to be inadequate, and the line of thought suggested above can be seen to be a powerful one. The crucial consideration is that the concept of necessity is meta-linguistic (or better, meta-conceptual), thus requiring that some concepts have already been successfully introduced before it in turn can be explained.

Recall that for logical objectivists conceptually necessary truths reflect objective relations between concepts (senses). It is but a trivial step from this to the idea that a necessary truth is *about* (involving reference to) concepts. Then a truth such as 'Necessarily nothing can be simultaneously red and green all over' should properly be construed as saying something like 'The concepts *red* and *green* mutually exclude one another.' How then could someone come to possess the concept of necessity – involving as it does reference to concepts – unless they already possessed some concepts to refer to? It would surely be viciously circular to claim that a notion involving reference to concepts could itself be presupposed to a grasp of those concepts themselves. In which case some concepts at least must be apprehensible prior to any grasp of the notion of necessity; which is as much as to say that there has to be a class of mutually independent elementary propositions.

It might be objected against this line of thought that, powerful or not, it cannot be correct to attribute it to Wittgenstein. For of course the *TLP* view is that necessity is not a genuine (*Sinn*-determining) concept, but is rather a formal one. Since the status of a proposition as necessary is a

feature of that proposition itself, his claim is that the attempt to assert its necessity will only result in nonsense (4.124, 4.125). How then can his main concern have been to provide a reductive account of necessity, if he believes that there is in reality no such concept? How can the point of the independence requirement have been to enable us to fix the sense of 'necessary' in a non-circular manner, if he thinks that the term is in fact nonsensical?

Although strictly correct, these points fail to undermine the interpretation. The official *TLP* doctrine of necessity-as-nonsense should no more prevent us from seeing Wittgenstein as trying to provide an account of the notion, than the official line that 'proposition', too, is a formal concept should stop us seeing him as trying to characterise the general propositional form (see 6). For the official doctrine merely reflects his account of what it is to say something, which is in fact violated systematically throughout *TLP* – and rightly so, as I argued briefly in Chapter 1.

(D) Doing without independence

We have now sketched two different arguments for the independence requirement. The first is that it must be possible to provide an articulation of that in virtue of which two incompatible concepts exclude one another, coupled with the plausible thought that such an account must in the end be truth-functional in form. The second is that it must be possible to provide a non-circular explanation of the notion of necessity, thus requiring us to generate a description of the set of all possible worlds without taking any modal notions for granted. Yet clearly there would have to be something wrong with each of these arguments if, as seems possible, the independence requirement cannot be complied with. Our immediate task is to find out what.

Consider first of all our second argument, from the possibility of reductive explanation. In fact this ultimately turns on the *TLP* account of the essence of communication, and of what it is to say something. For the only reason we have for denying that it is possible to explain the concept of necessity to someone on the basis of their grasp of a set of propositions which are not logically independent is that the idea of necessity is already presupposed in (entailed by) what they know. Recall that on the *TLP* account of semantic content 'Nothing is red and green all over' would say the very same as 'Necessarily: nothing is red and green all over' (because they are logically equivalent). That is why one could not explain the latter

on the basis of an understanding of the former: they say the very same thing. But if we reject the ubiquity of this account of semantic content, as I urged in Chapter 1 that we should, then the argument collapses. We can allow that necessity is meta-conceptual, involving reference to some already-existing concepts, while still explaining it to someone on the basis of their grasp of concepts which necessarily exclude one another. For lacking as yet any explicit notion of necessity, they will not have realised that this is so. Indeed, it is precisely this that our explanation would bring them to see.⁸

What, then, of our first argument: the insistence that the grounds for any necessary truth be articulable? Here we need to consider what account might be given of the necessity of colour-exclusion, supposing that sentences such as 'This is red' and 'This is green' were genuinely elementary. As we mentioned earlier, on his return to philosophy in the late twenties Wittgenstein put forward an account according to which applying any one colour concept involves the application of all; individual colour concepts being like the marks on a ruler. Then in the same way that laying a ruler against an object and seeing that it is a particular length automatically implies that it is not any of the other lengths, so laying the whole colour system against an object, and judging that it is red, implies automatically that it is not green. Thus the idea is that by showing the colour concepts to be essentially part of a colour system, or colour space, we can exhibit the grounds for the necessity of colour-exclusion without having to subject the individual colour concepts to analysis.

Even if we grant Wittgenstein what is in fact far from obvious – that the understanding of any single colour concept presupposes an understanding of all⁹ – the proposed solution will not work. For all it does is to explain colour-exclusion in terms of spatial-exclusion ('a boundary cannot be at two places on a ruler at once'). But this is equally problematic, if sentences ascribing spatial positions to objects are regarded as elementary. For in virtue of what is it necessary that an object cannot be in two distinct places at once? How are we to exhibit this necessity as flowing from the nature of the concepts involved? Of course one might reply that spatial position is the criterion for individuating physical objects; and this is no doubt correct. But then it is precisely to say that the concept of an individual object is analysable in such a way as to explain the necessity of 'a given object cannot be in two distinct places at once'. And there is no prospect of finding within that analysis a parallel principle for colours, since it is of course possible for a physical object to have no colour (to be transparent). So to say that the colour concepts are individually unanalysable, but are

essentially members of a system of such concepts, is as yet to provide no explanation for why a surface cannot have two colours at once.

I can see no way for Wittgenstein to maintain both that there are no purely metaphysical necessities, and that necessity is objective, if it is allowed that there can be mutually inconsistent elementary propositions. Yet each is an essential ingredient in the philosophy of *TLP*. Moreover, the rejection of metaphysical intuition surely stands firm in any case. As already indicated in Chapter 3, I agree with Wittgenstein in finding the idea that our intellects might be such as to provide us with direct access to the metaphysical structure of reality wholly unintelligible. So it looks as if our only option is to give up the objectivity of conceptual necessity – in effect denying logical objectivism. Our account of the necessity of ‘Nothing is red and green all over’ could then take the sort of form we began to indicate at the end of Chapter 3, somewhat as follows. Having acquired the concepts *red* and *green*, we discover that we find it unimaginable how any surface could satisfy both concepts at once. Now so far this is merely a fact of psychology. But as a result of it we elevate, by convention, the proposition in question to the status of a necessary ‘truth’ (we ‘put it in the archives’), employing it henceforward as a rule of description.¹⁰

Summary

We have found two distinct arguments for the independence requirement. One is premised on the reducibility, and one on the articulability, of necessity. Each may ultimately be unsound. But the reasons for their failure would lie deep within the structure of *TLP*: respectively, in its account of the identity-conditions for semantic content, and in its logical objectivism.

14 Modelling elementary propositions

Our task in this chapter is to provide a model for the elementary propositions of *TLP* which will meet as many as possible of the various constraints on their nature to have emerged from our previous discussions.

(A) Constraints on the model

We can distinguish three distinct kinds of constraint which a completely satisfactory model should meet. Firstly, there are Wittgenstein’s explicit claims, which are so clear as to admit of no reinterpretation. No model could even begin to be adequate which did not respect them. We are told that Simples exist in all possible worlds (2.023), that elementary propositions consist only of names of Simples in immediate concatenation (3.2–3.21, 4.22), that elementary propositions are logically independent of one another (4.211, 5.134), and that an assignment of truth-values to all elementary propositions would constitute a complete description of the world (4.26). Many commentators have thought that nothing could even begin to satisfy all of these constraints. We shall show in the next section that they are wrong.

Secondly, there are the further developments of the above claims which are imposed upon us by some of the interpretations adopted earlier. We argued that the Simples of *TLP* are individuals, as opposed to universals (Chapter 1), which not only exist in all possible worlds but at all times in those worlds (Chapter 8). So the Simples in our model should be necessarily existing individuals. Moreover, since this narrow reading of ‘object’ requires an equally narrow reading of ‘name’, it will follow that elementary propositions consist only of proper names in immediate concatenation. So unless we were prepared to countenance Sellars’ suggestion in his (1962a), that *TLP* allows monadic concatenations (propositions consisting of just a single name), it would be an advantage if our model were